Statistical Natural Language Processing

N-gram Language Models

Çağrı Çöltekin

University of Tübingen
Seminar für Sprachwissenschaft

Summer Semester 2021

version: a672aaa+ @2021-07-21

N-gram language models

- A language model answers the question *how likely is a sequence of words in a given language?*
- They assign scores, typically probabilities, to sequences (of words, letters, …)
- n-gram language models are the 'classical' approach to language modeling
- The main idea is to estimate probabilities of sequences, using the probabilities of words given a limited history
- As a bonus we get the answer for *what is the most likely word given previous words?*

N-grams in practice: spelling correction

- How would a spell checker know that there is a spelling error in the following sentence?

  I like pizza wit spinach

- Or this one?

  Zoo animals on the lose

N-grams in practice: spelling correction

- How would a spell checker know that there is a spelling error in the following sentence?

  I like pizza wit spinach

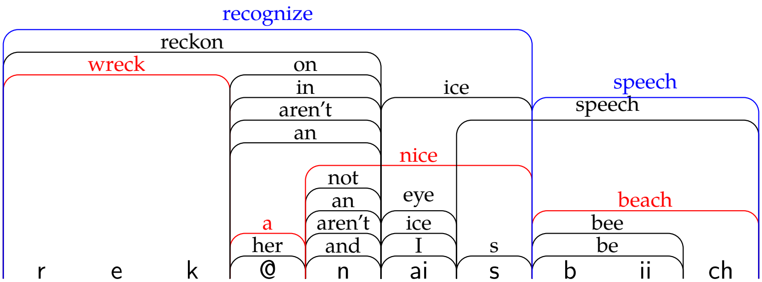
- Or this one?

  Zoo animals on the lose

We want:

$$\begin{aligned} P(\text{I like pizza with spinach}) &> P(\text{I like pizza wit spinach}) \\ P(\text{Zoo animals on the loose}) &> P(\text{Zoo animals on the lose}) \end{aligned}$$

N-grams in practice: speech recognition



We want:

$$P(\text{recognize speech}) > P(\text{wreck a nice beach})$$

* Reproduced from Shillcock (1995)

More applications for language models

- Spelling correction
- Speech recognition
- Machine translation
- Predictive text
- Text recognition (OCR, handwritten)
- Information retrieval
- Question answering
- Text classification
- In general, pre-trained (neural) language models can bring additional linguistic/world knowledge to almost any NLP task

Our aim

We want to solve two related problems:

- Given a sequence of words $\boldsymbol{w} = (w_1 w_2 \ldots w_m)$,

  what is the probability of the sequence $P(\boldsymbol{w})$?

  (machine translation, automatic speech recognition, spelling correction)

- Given a sequence of words $w_1 w_2 \ldots w_{m-1}$,

  what is the probability of the next word $P(w_m \mid w_1 \ldots w_{m-1})$?

  (predictive text)

Assigning probabilities to sentences

count and divide?

How do we calculate the probability of a sentence like

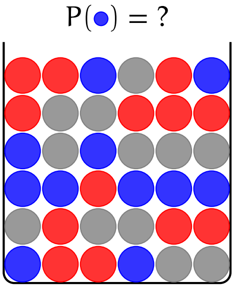
$$P(\text{I like pizza with spinach})$$

Assigning probabilities to sentences

count and divide?

How do we calculate the probability of a sentence like
P(I like pizza with spinach)

- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?

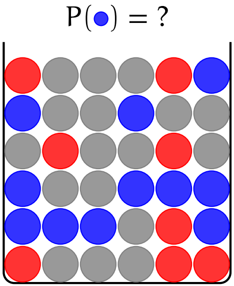


Assigning probabilities to sentences

count and divide?

How do we calculate the probability of a sentence like
P(I like pizza with spinach)

- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?
- Short answer: No.

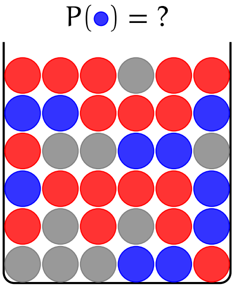


Assigning probabilities to sentences

count and divide?

How do we calculate the probability of a sentence like
P(I like pizza with spinach)

- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?
- Short answer: No.
  - Many sentences are not observed even in very large corpora

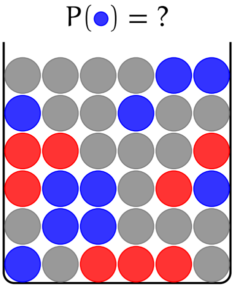


Assigning probabilities to sentences

count and divide?

How do we calculate the probability of a sentence like
P(I like pizza with spinach)

- Can we count the occurrences of the sentence, and divide it by the total number of sentences (in a large corpus)?
- Short answer: No.
  - Many sentences are not observed even in very large corpora
  - For the ones observed in a corpus, probabilities will not reflect our intuitions, or will not be useful in most applications



Assigning probabilities to sentences

applying the chain rule

- The solution is to *decompose*

  We use probabilities of parts of the sentence (words) to calculate the probability of the whole sentence

- Using the chain rule of probability (without loss of generality), we can write

$$
\begin{aligned}
P(w_1, w_2, \ldots, w_m) = \; & P(w_2 \mid w_1) \\
& \times P(w_3 \mid w_1, w_2) \\
& \times \ldots \\
& \times P(w_m \mid w_1, w_2, \ldots w_{m-1})
\end{aligned}
$$

Example: applying the chain rule

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \; & P(\text{like} \mid \text{I}) \\
\times \; & P(\text{pizza} \mid \text{I like}) \\
\times \; & P(\text{with} \mid \text{I like pizza}) \\
\times \; & P(\text{spinach} \mid \text{I like pizza with})
\end{aligned}
$$

- Did we solve the problem?

Example: applying the chain rule

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \;& P(\text{like} \mid \text{I}) \\
\times\;& P(\text{pizza} \mid \text{I like}) \\
\times\;& P(\text{with} \mid \text{I like pizza}) \\
\times\;& P(\text{spinach} \mid \text{I like pizza with})
\end{aligned}
$$

- Did we solve the problem?
- Not really, the last term is equally difficult to estimate

Example: bigram probabilities of a sentence

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \; & P(\text{like} \mid \text{I}) \\
& \times P(\text{pizza} \mid \text{I like}) \\
& \times P(\text{with} \mid \text{I like pizza}) \\
& \times P(\text{spinach} \mid \text{I like pizza with})
\end{aligned}
$$

Example: bigram probabilities of a sentence

with first-order Markov assumption

$$
\begin{aligned}
P(\text{I like pizza with spinach}) = \; & P(\text{like} \mid \text{I}) \\
\times \; & P(\text{pizza} \mid \text{like}) \\
\times \; & P(\text{with} \mid \text{pizza}) \\
\times \; & P(\text{spinach} \mid \text{with})
\end{aligned}
$$

- Now, hopefully, we can count them in a corpus

Maximum-likelihood estimation (MLE)

- The MLE of n-gram probabilities is based on their frequencies in a corpus
- We are interested in conditional probabilities of the form: $P(w_i \mid w_1, \ldots, w_{i-1})$, which we estimate using

$$P(w_i \mid w_{i-n+1}, \ldots, w_{i-1}) = \frac{C(w_{i-n+1} \ldots w_i)}{C(w_{i-n+1} \ldots w_{i-1})}$$

  where, $C()$ is the frequency (count) of the sequence in the corpus.
- For example, the probability $P(\text{like} \mid \text{I})$ would be

$$\begin{aligned} P(\text{like} \mid \text{I}) &= \frac{C(\text{I like})}{C(\text{I})} \\ &= \frac{\text{number of times I like occurs in the corpus}}{\text{number of times I occurs in the corpus}} \end{aligned}$$

MLE estimation of an n-gram language model

An n-gram model conditioned on $n - 1$ previous words.

$$
\begin{aligned}
&\text{unigram} & P(w_i) &= \frac{C(w_i)}{N} \\
&\text{bigram} & P(w_i) = P(w_i \mid w_{i-1}) &= \frac{C(w_{i-1}w_i)}{C(w_{i-1})} \\
&\text{trigram} & P(w_i) = P(w_i \mid w_{i-2}w_{i-1}) &= \frac{C(w_{i-2}w_{i-1}w_i)}{C(w_{i-2}w_{i-1})}
\end{aligned}
$$

Parameters of an n-gram model are these conditional probabilities.

Unigrams

Unigrams are simply the single words (or tokens).

A small corpus

I'm sorry, Dave.
I'm afraid I can't do that.

Unigrams

Unigrams are simply the single words (or tokens).

A small corpus

I 'm sorry , Dave .
I 'm afraid I can 't do that .

When tokenized, we have 15 *tokens*, and 11 *types*.

Unigram counts

I	3	,	1	afraid	1	do	1
'm	2	Dave	1	can	1	that	1
sorry	1	.	2	't	1		

Traditionally, *can't* is tokenized as *ca␣n't* (similar to *have␣n't*, *is␣n't* etc.), but for our purposes *can␣'t* is more readable.

Unigram probability of a sentence

Unigram counts

I	3	,	1	afraid	1	do	1
'm	2	Dave	1	can	1	that	1
sorry	1	.	2	't	1		

$$
\begin{aligned}
&\mathtt{P(I\ 'm\ sorry\ ,\ Dave\ .)} \\
&= \mathtt{P(I)} \times \mathtt{P('m)} \times \mathtt{P(sorry)} \times \mathtt{P(,)} \times \mathtt{P(Dave)} \times \mathtt{P(.)} \\
&= \frac{3}{15} \times \frac{2}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{2}{15} \\
&= 0.000\,001\,05
\end{aligned}
$$

Unigram probability of a sentence

Unigram counts

I	3	,	1	afraid	1	do	1
'm	2	Dave	1	can	1	that	1
sorry	1	.	2	't	1		

$$\begin{aligned}
&\texttt{P(I 'm sorry , Dave .)} \\
&= \texttt{P(I)} \times \texttt{P('m)} \times \texttt{P(sorry)} \times \texttt{P(,)} \times \texttt{P(Dave)} \times \texttt{P(.)} \\
&= \frac{3}{15} \times \frac{2}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{2}{15} \\
&= 0.000\,001\,05
\end{aligned}$$

- $\texttt{P(, 'm I . sorry Dave)} = ?$
- Where did all the probability mass go?
- What is the most likely sentence according to this model?

N-gram models define probability distributions

- An n-gram model defines a probability distribution over words

$$\sum_{w \in V} P(w) = 1$$

- They also define probability distributions over word sequences of equal size. For example (length 2),

$$\sum_{w \in V} \sum_{v \in V} P(w)P(v) = 1$$

word	prob
I	0.200
'm	0.133
.	0.133
't	0.067
,	0.067
Dave	0.067
afraid	0.067
can	0.067
do	0.067
sorry	0.067
that	0.067
	1.000

N-gram models define probability distributions

- An n-gram model defines a probability distribution over words

$$\sum_{w \in V} P(w) = 1$$

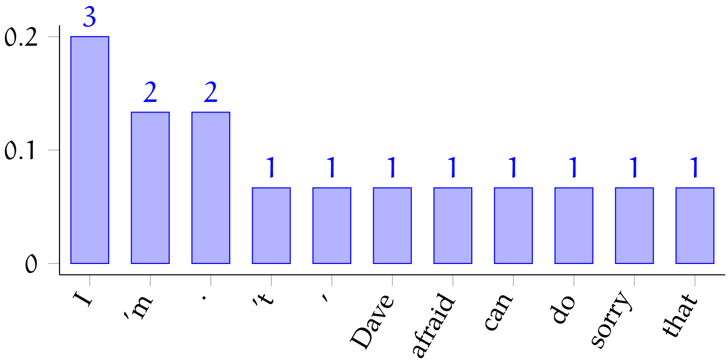
- They also define probability distributions over word sequences of equal size. For example (length 2),

$$\sum_{w \in V} \sum_{v \in V} P(w)P(v) = 1$$

- What about sentences?

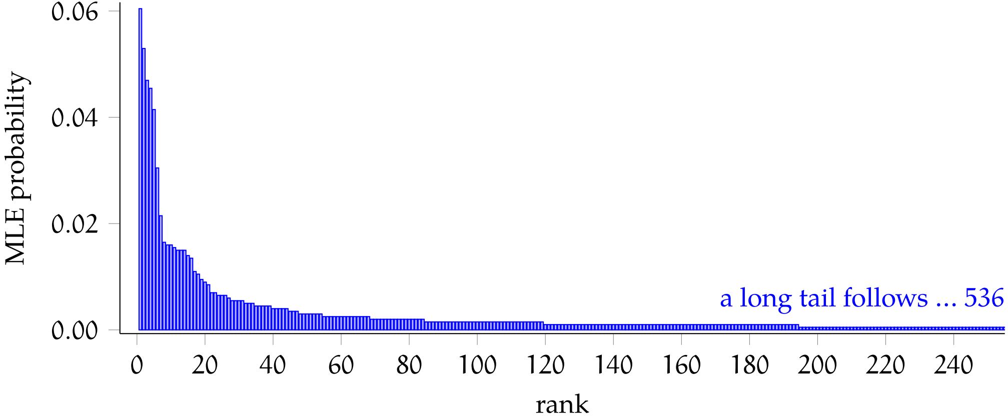
word	prob
I	0.200
'm	0.133
.	0.133
't	0.067
,	0.067
Dave	0.067
afraid	0.067
can	0.067
do	0.067
sorry	0.067
that	0.067
	1.000

Unigram probabilities



Unigram probabilities in a (slightly) larger corpus

MLE probabilities in the Universal Declaration of Human Rights



Zipf's law – a short divergence

The frequency of a word is inversely proportional to its rank:

$$\text{rank} \times \text{frequency} = \text{k} \quad \text{or} \quad \text{frequency} \propto \frac{1}{\text{rank}}$$

- This is a reoccurring theme in (computational) linguistics: most linguistic units follow more-or-less a similar distribution
- Important consequence for us (in this lecture):
  - even very large corpora will *not* contain some of the words (or n-grams)
  - there will be many low-probability events (words/n-grams)

Bigrams

Bigrams are overlapping sequences of two tokens.

I 'm sorry , Dave .

I 'm afraid I can 't do that .

Bigram counts

ngram	freq	ngram	freq	ngram	freq	ngram	freq
I 'm	2	, Dave	1	afraid I	1	n't do	1
'm sorry	1	Dave .	1	I can	1	do that	1
sorry ,	1	'm afraid	1	can 't	1	that .	1

Bigrams

Bigrams are overlapping sequences of two tokens.

I 'm sorry , Dave .

I 'm afraid I can 't do that .

Bigram counts

ngram	freq	ngram	freq	ngram	freq	ngram	freq
I 'm	2	, Dave	1	afraid I	1	n't do	1
'm sorry	1	Dave .	1	I can	1	do that	1
sorry ,	1	'm afraid	1	can 't	1	that .	1

- What about the bigram ' `. I` '?

Sentence boundary markers

If we want sentence probabilities, we need to mark them.

$\langle s \rangle$ I 'm sorry , Dave . $\langle /s \rangle$
$\langle s \rangle$ I 'm afraid I can 't do that . $\langle /s \rangle$

- The bigram '$\langle s \rangle$ `I`' is not the same as the unigram '`I`'
  Including $\langle s \rangle$ allows us to predict likely words at the beginning of a sentence
- Including $\langle /s \rangle$ allows us to assign a proper probability distribution to sentences

Calculating bigram probabilities

recap with some more detail

We want to calculate $P(w_2 \mid w_1)$. From the chain rule:

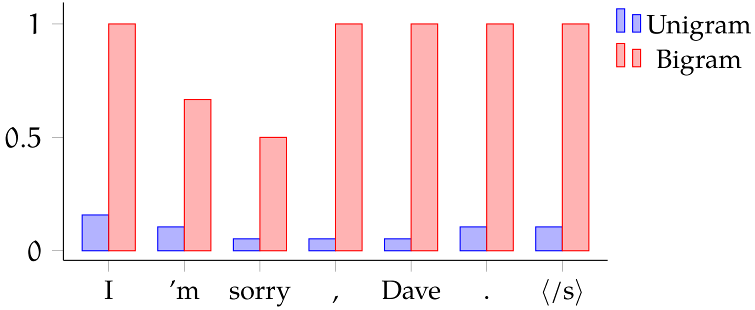
$$P(w_2 \mid w_1) = \frac{P(w_1, w_2)}{P(w_1)}$$

and, the MLE

$$P(w_2 \mid w_1) = \frac{\frac{C(w_1 w_2)}{N}}{\frac{C(w_1)}{N}} = \frac{C(w_1 w_2)}{C(w_1)}$$

$P(w_2 \mid w_1)$	is the probability of w_2 given the previous word is w_1
$P(w_1, w_2)$	is the probability of the sequence $w_1 w_2$
$P(w_1)$	is the probability of w_1 occurring as the first item in a bigram, not its unigram probability

Sentence probability: bigram vs. unigram



$$P_{\text{uni}}(\langle s\rangle \text{ I 'm sorry , Dave . } \langle /s\rangle) = 2.83 \times 10^{-9}$$
$$P_{\text{bi}}(\langle s\rangle \text{ I 'm sorry , Dave . } \langle /s\rangle) = 0.33$$

Unigram vs. bigram probabilities

in sentences and non-sentences

w	I	'm	sorry	,	Dave	.	
P_{uni}	0.18	0.12	0.06	0.06	0.06	0.12	4.97×10^{-7}
P_{bi}	1.00	0.67	0.50	1.00	1.00	1.00	0.33

Unigram vs. bigram probabilities

in sentences and non-sentences

w	I	'm	sorry	,	Dave	.	
P_{uni}	0.18	0.12	0.06	0.06	0.06	0.12	4.97×10^{-7}
P_{bi}	1.00	0.67	0.50	1.00	1.00	1.00	0.33

w	,	'm	I	.	sorry	Dave	
P_{uni}	0.06	0.12	0.18	0.12	0.06	0.06	4.97×10^{-7}
P_{bi}	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Unigram vs. bigram probabilities

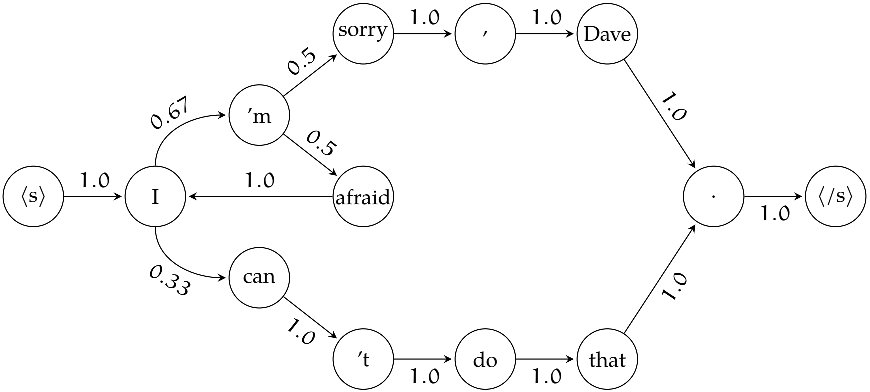
in sentences and non-sentences

w	I	'm	sorry	,	Dave	.	
P_{uni}	0.18	0.12	0.06	0.06	0.06	0.12	4.97×10^{-7}
P_{bi}	1.00	0.67	0.50	1.00	1.00	1.00	0.33

w	,	'm	I	.	sorry	Dave	
P_{uni}	0.06	0.12	0.18	0.12	0.06	0.06	4.97×10^{-7}
P_{bi}	0.00	0.00	0.00	0.00	0.00	0.00	0.00

w	I	'm	afraid	,	Dave	.	
P_{uni}	0.18	0.12	0.06	0.06	0.06	0.12	4.97×10^{-7}
P_{bi}	1.00	0.67	0.50	0.00	1.00	1.00	0.00

Bigram models as weighted finite-state automata



Trigrams

⟨s⟩ ⟨s⟩ I ’m sorry , Dave . ⟨/s⟩
⟨s⟩ ⟨s⟩ I ’m afraid I can ’t do that . ⟨/s⟩

Trigram counts

ngram	freq	ngram	freq	ngram	freq
⟨s⟩ ⟨s⟩ I	2	do that .	1	that . ⟨/s⟩	1
⟨s⟩ I ’m	2	I ’m sorry	1	’m sorry ,	1
sorry , Dave	1	, Dave .	1	Dave . ⟨/s⟩	1
I ’m afraid	1	’m afraid I	1	afraid I can	1
I can ’t	1	can ’t do	1	’t do that	1

Trigrams

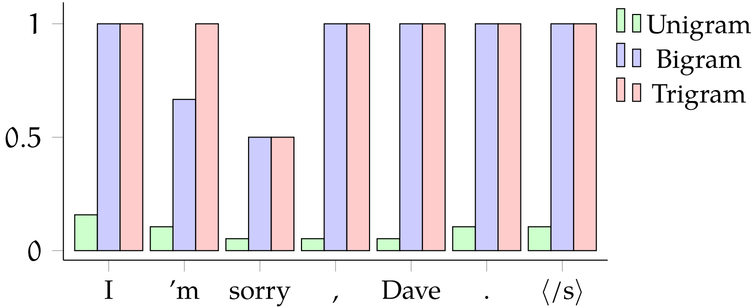
$\langle s \rangle$ $\langle s \rangle$ I 'm sorry , Dave . $\langle /s \rangle$
$\langle s \rangle$ $\langle s \rangle$ I 'm afraid I can 't do that . $\langle /s \rangle$

Trigram counts

ngram	freq	ngram	freq	ngram	freq
$\langle s \rangle$ $\langle s \rangle$ I	2	do that .	1	that . $\langle /s \rangle$	1
$\langle s \rangle$ I 'm	2	I 'm sorry	1	'm sorry ,	1
sorry , Dave	1	, Dave .	1	Dave . $\langle /s \rangle$	1
I 'm afraid	1	'm afraid I	1	afraid I can	1
I can 't	1	can 't do	1	't do that	1

- How many n-grams are there in a sentence of length m?

Trigram probabilities of a sentence



$$P_{uni}(\text{I 'm sorry , Dave . } \langle/\text{s}\rangle) = 2.83 \times 10^{-9}$$
$$P_{bi}(\text{I 'm sorry , Dave . } \langle/\text{s}\rangle) = 0.33$$
$$P_{tri}(\text{I 'm sorry , Dave . } \langle/\text{s}\rangle) = 0.50$$

Short detour: colorless green ideas

But it must be recognized that the notion 'probability of a sentence' is an entirely useless one, under any known interpretation of this term. — Chomsky (1968)

- The following 'sentences' are categorically different:
  - Furiously sleep ideas green colorless
  - Colorless green ideas sleep furiously

Short detour: colorless green ideas

But it must be recognized that the notion 'probability of a sentence' is an entirely useless one, under any known interpretation of this term. — Chomsky (1968)

- The following 'sentences' are categorically different:
  - Furiously sleep ideas green colorless
  - Colorless green ideas sleep furiously
- Can n-gram models model the difference?

Short detour: colorless green ideas

But it must be recognized that the notion 'probability of a sentence' is an entirely useless one, under any known interpretation of this term. — Chomsky (1968)

- The following 'sentences' are categorically different:
  - Furiously sleep ideas green colorless
  - Colorless green ideas sleep furiously
- Can n-gram models model the difference?
- Should n-gram models model the difference?

What do n-gram models model?

- Some morphosyntax: the bigram '`ideas are`' is (much) more likely than '`ideas is`'

What do n-gram models model?

- Some morphosyntax: the bigram '`ideas are`' is (much) more likely than '`ideas is`'
- Some semantics: '`bright ideas`' is more likely than '`green ideas`'

What do n-gram models model?

- Some morphosyntax: the bigram '`ideas are`' is (much) more likely than '`ideas is`'
- Some semantics: '`bright ideas`' is more likely than '`green ideas`'
- Some cultural aspects of everyday language: '`Chinese food`' is more likely than '`British food`'

What do n-gram models model?

- Some morphosyntax: the bigram '`ideas are`' is (much) more likely than '`ideas is`'
- Some semantics: '`bright ideas`' is more likely than '`green ideas`'
- Some cultural aspects of everyday language: '`Chinese food`' is more likely than '`British food`'
- more aspects of 'usage' of language

How to test n-gram models?

Extrinsic: improvement of the target application due to the language model:

- Speech recognition accuracy
- BLEU score for machine translation
- Keystroke savings in predictive text applications

Intrinsic: the higher the probability assigned to a test set better the model. A few measures:

- Likelihood
- (cross) entropy
- perplexity

How to test n-gram models?

Extrinsic: improvement of the target application due to the language model:

- Speech recognition accuracy
- BLEU score for machine translation
- Keystroke savings in predictive text applications

Intrinsic: the higher the probability assigned to a test set better the model. A few measures:

- Likelihood
- (cross) entropy
- perplexity

Like any ML method, test set has to be different than training set.

Intrinsic evaluation metrics: likelihood

- Likelihood of a model M is the probability of the (test) set $\boldsymbol{w}$ given the model

$$\mathcal{L}(M \mid \boldsymbol{w}) = P(\boldsymbol{w} \mid M) = \prod_{s \in \boldsymbol{w}} P(s)$$

Intrinsic evaluation metrics: likelihood

- Likelihood of a model M is the probability of the (test) set $\boldsymbol{w}$ given the model

$$\mathcal{L}(M \mid \boldsymbol{w}) = P(\boldsymbol{w} \mid M) = \prod_{s \in \boldsymbol{w}} P(s)$$

- The higher the likelihood (for a given test set), the better the model

Intrinsic evaluation metrics: likelihood

- Likelihood of a model M is the probability of the (test) set $\boldsymbol{w}$ given the model

$$\mathcal{L}(M \mid \boldsymbol{w}) = P(\boldsymbol{w} \mid M) = \prod_{s \in \boldsymbol{w}} P(s)$$

- The higher the likelihood (for a given test set), the better the model
- Likelihood is sensitive to the test set size

Intrinsic evaluation metrics: likelihood

- Likelihood of a model M is the probability of the (test) set $\boldsymbol{w}$ given the model

$$\mathcal{L}(M \mid \boldsymbol{w}) = P(\boldsymbol{w} \mid M) = \prod_{s \in \boldsymbol{w}} P(s)$$

- The higher the likelihood (for a given test set), the better the model
- Likelihood is sensitive to the test set size
- Practical note: (minus) log likelihood is used more commonly, because of ease of numerical manipulation

Intrinsic evaluation metrics: cross entropy

- Cross entropy of a language model on a test set $\boldsymbol{w}$ is

$$H(\boldsymbol{w}) = -\frac{1}{N} \sum_{w_i} \log_2 \widehat{P}(w_i)$$

Intrinsic evaluation metrics: cross entropy

- Cross entropy of a language model on a test set $\boldsymbol{w}$ is

$$H(\boldsymbol{w}) = -\frac{1}{N} \sum_{w_i} \log_2 \widehat{P}(w_i)$$

- The lower the cross entropy, the better the model

Intrinsic evaluation metrics: cross entropy

- Cross entropy of a language model on a test set $\boldsymbol{w}$ is

$$H(\boldsymbol{w}) = -\frac{1}{N} \sum_{w_i} \log_2 \widehat{P}(w_i)$$

- The lower the cross entropy, the better the model
- Cross entropy is not sensitive to the test-set size

Intrinsic evaluation metrics: cross entropy

- Cross entropy of a language model on a test set $\boldsymbol{w}$ is

$$H(\boldsymbol{w}) = -\frac{1}{N} \sum_{w_i} \log_2 \widehat{P}(w_i)$$

- The lower the cross entropy, the better the model
- Cross entropy is not sensitive to the test-set size

Intrinsic evaluation metrics: cross entropy

- Cross entropy of a language model on a test set $\boldsymbol{w}$ is

$$H(\boldsymbol{w}) = -\frac{1}{N} \sum_{w_i} \log_2 \widehat{P}(w_i)$$

- The lower the cross entropy, the better the model
- Cross entropy is not sensitive to the test-set size

Reminder: Cross entropy is the bits required to encode the data coming from P using another (approximate) distribution $\widehat{P}$.

$$H(P, Q) = -\sum_{x} P(x) \log \widehat{P}(x)$$

Intrinsic evaluation metrics: perplexity

- Perplexity is a more common measure for evaluating language models

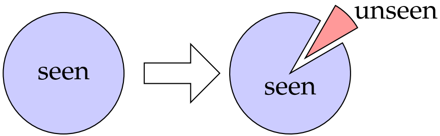
$$\mathrm{PP}(\boldsymbol{w}) = 2^{\mathrm{H}(\boldsymbol{w})} = \mathrm{P}(\boldsymbol{w})^{-\frac{1}{N}} = \sqrt[N]{\frac{1}{\mathrm{P}(\boldsymbol{w})}}$$

- Perplexity is the average branching factor
- Similar to cross entropy
  - lower better
  - not sensitive to test set size

What do we do with unseen n-grams?

...and other issues with MLE estimates

- Words (and word sequences) are distributed according to the Zipf's law: *many words are rare*.
- MLE will assign 0 probabilities to unseen words, and sequences containing unseen words
- Even with non-zero probabilities, MLE *overfits* the training data
- One solution is smoothing: take some probability mass from known words, and assign it to unknown words



Laplace smoothing

(Add-one smoothing)

- The idea (from 1790): add one to all counts
- The probability of a word is estimated by

$$P_{+1}(w) = \frac{C(w)+1}{N+V}$$

N number of word tokens
V number of word types - the size of the vocabulary

- Then, probability of an unknown word is:

$$\frac{0+1}{N+V}$$

Laplace smoothing

for n-grams

- The probability of a bigram becomes

$$P_{+1}(w_i w_{i-1}) = \frac{C(w_i w_{i-1}) + 1}{N + V^2}$$

- and, the conditional probability

$$P_{+1}(w_i \mid w_{i-1}) = \frac{C(w_{i-1} w_i) + 1}{C(w_{i-1}) + V}$$

- In general

$$P_{+1}(w_{i-n+1}^{i}) = \frac{C(w_{i-n+1}^{i}) + 1}{N + V^n}$$

$$P_{+1}(w_{i-n+1}^{i} \mid w_{i-n+1}^{i-1}) = \frac{C(w_{i-n+1}^{i}) + 1}{C(w_{i-n+1}^{i-1}) + V}$$

Bigram probabilities

MLE vs. Laplace smoothing

$w_1 w_2$	C_{+1}	$P_{MLE}(w_1 w_2)$	$P_{+1}(w_1 w_2)$	$P_{MLE}(w_2 \mid w_1)$	$P_{+1}(w_2 \mid w_1)$
⟨s⟩ I	3	0.118	0.019	1.000	0.188
I 'm	3	0.118	0.019	0.667	0.176
'm sorry	2	0.059	0.012	0.500	0.125
sorry ,	2	0.059	0.012	1.000	0.133
, Dave	2	0.059	0.012	1.000	0.133
Dave .	2	0.059	0.012	1.000	0.133
'm afraid	2	0.059	0.012	0.500	0.125
afraid I	2	0.059	0.012	1.000	0.133
I can	2	0.059	0.012	0.333	0.118
can 't	2	0.059	0.012	1.000	0.133
n't do	2	0.059	0.012	1.000	0.133
do that	2	0.059	0.012	1.000	0.133
that .	2	0.059	0.012	1.000	0.133
. ⟨/s⟩	3	0.118	0.019	1.000	0.188
$\sum$		1.000	0.193		

MLE vs. Laplace probabilities

probabilities of sentences and non-sentences (based on the bigram model)

w	I	'm	sorry	,	Dave	.	⟨/s⟩	
P_{MLE}	1.00	0.67	0.50	1.00	1.00	1.00	1.00	0.33
P_{+1}	0.19	0.18	0.13	0.13	0.13	0.13	0.19	1.84×10^{-6}

MLE vs. Laplace probabilities

probabilities of sentences and non-sentences (based on the bigram model)

w	I	'm	sorry	,	Dave	.	⟨/s⟩	
P_{MLE}	1.00	0.67	0.50	1.00	1.00	1.00	1.00	0.33
P_{+1}	0.19	0.18	0.13	0.13	0.13	0.13	0.19	1.84×10^{-6}

w	,	'm	I	.	sorry	Dave	⟨/s⟩	
P_{MLE}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P_{+1}	0.03	0.03	0.03	0.03	0.03	0.03	0.03	1.17×10^{-12}

MLE vs. Laplace probabilities

probabilities of sentences and non-sentences (based on the bigram model)

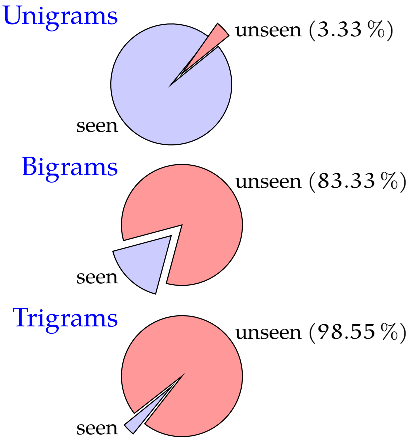
w	I	'm	sorry	,	Dave	.	$\langle/s\rangle$	
P_{MLE}	1.00	0.67	0.50	1.00	1.00	1.00	1.00	0.33
P_{+1}	0.19	0.18	0.13	0.13	0.13	0.13	0.19	1.84×10^{-6}

w	,	'm	I	.	sorry	Dave	$\langle/s\rangle$	
P_{MLE}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P_{+1}	0.03	0.03	0.03	0.03	0.03	0.03	0.03	1.17×10^{-12}

w	I	'm	afraid	,	Dave	.	$\langle/s\rangle$	
P_{MLE}	1.00	0.67	0.50	0.00	1.00	1.00	1.00	0.00
P_{+1}	0.19	0.18	0.13	0.03	0.13	0.13	0.19	4.45×10^{-7}

How much probability mass does +1 smoothing steal?

- Laplace smoothing reserves probability mass proportional to the size of the vocabulary
- This is just too much for large vocabularies and higher order n-grams
- Note that only very few of the higher level n-grams (e.g., trigrams) are possible



Lidstone correction

(Add-α smoothing)

- A simple improvement over Laplace smoothing is adding α instead of 1

$$P_{+\alpha}(w_{i-n+1}^{i} \mid w_{i-n+1}^{i-1}) = \frac{C(w_{i-n+1}^{i}) + \alpha}{C(w_{i-n+1}^{i-1}) + \alpha V}$$

- With smaller α values, the model behaves similar to MLE, it overfits: it has high variance
- Larger α values reduce overfitting/variance, but result in large bias

Lidstone correction

(Add-α smoothing)

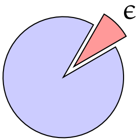
- A simple improvement over Laplace smoothing is adding α instead of 1

$$P_{+\alpha}(w_{i-n+1}^{i} \mid w_{i-n+1}^{i-1}) = \frac{C(w_{i-n+1}^{i}) + \alpha}{C(w_{i-n+1}^{i-1}) + \alpha V}$$

- With smaller α values, the model behaves similar to MLE, it overfits: it has high variance
- Larger α values reduce overfitting/variance, but result in large bias

We need to tune α like any other hyperparameter.

Absolute discounting



- An alternative to the additive smoothing is to reserve an explicit amount of probability mass, ϵ, for the unseen events
- The probabilities of known events has to be re-normalized
- How do we decide what ϵ value to use?

Good-Turing smoothing

- Estimate the probability mass to be reserved for the novel n-grams using the observed n-grams
- Novel events in our training set is the ones that occur once

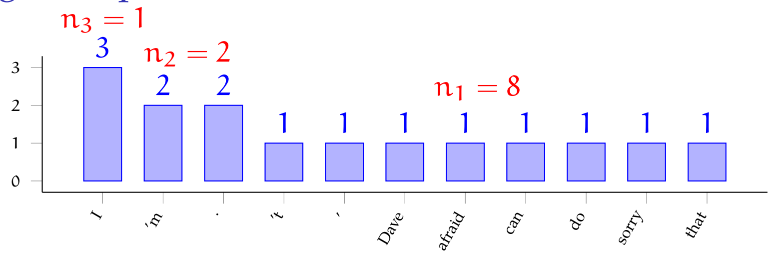
$$p_0 = \frac{n_1}{n}$$

where n_1 is the number of distinct n-grams with frequency 1 in the training data

- Now we need to discount this mass from the higher counts
- The probability of an n-gram that occurred r times in the corpus is

$$(r+1)\frac{n_{r+1}}{n_r n}$$

Good-Turing example



$$P_{GT}(\text{the}) + P_{GT}(\text{a}) + \ldots = \frac{8}{15}$$

$$P_{GT}(\text{that}) = P_{GT}(\text{do}) = \ldots = \frac{2 \times 2}{15 \times 8}$$

$$P_{GT}(\text{'m}) = P_{GT}(.) = \frac{3 \times 1}{15 \times 2}$$

Issues with Good-Turing discounting

With some solutions

- Zero counts: we cannot assign probabilities if $n_{r+1} = 0$
- The estimates of some of the frequencies of frequencies are unreliable
- A solution is to replace n_r with smoothed counts z_r
- A well-known technique (simple Good-Turing) for smoothing n_r is to use linear interpolation

$$\log z_r = a + b \log r$$

Not all (unknown) n-grams are equal

- Let's assume that `black squirrel` is an unknown bigram
- How do we calculate the smoothed probability

$$P_{+1}(\texttt{squirrel} \mid \texttt{black}) =$$

Not all (unknown) n-grams are equal

- Let's assume that `black squirrel` is an unknown bigram
- How do we calculate the smoothed probability

$$P_{+1}(\texttt{squirrel} \mid \texttt{black}) = \frac{0 + 1}{C(\texttt{black}) + V}$$

Not all (unknown) n-grams are equal

- Let's assume that `black squirrel` is an unknown bigram
- How do we calculate the smoothed probability

$$P_{+1}(\texttt{squirrel} \mid \texttt{black}) = \frac{0 + 1}{C(\texttt{black}) + V}$$

- How about `black wug`?

$$P_{+1}(\texttt{black wug}) =$$

Not all (unknown) n-grams are equal

- Let's assume that `black squirrel` is an unknown bigram
- How do we calculate the smoothed probability

$$P_{+1}(\texttt{squirrel} \mid \texttt{black}) = \frac{0+1}{C(\texttt{black}) + V}$$

- How about `black wug`?

$$P_{+1}(\texttt{black wug}) = P_{+1}(\texttt{wug} \mid \texttt{black}) =$$

Not all (unknown) n-grams are equal

- Let's assume that `black squirrel` is an unknown bigram
- How do we calculate the smoothed probability

$$P_{+1}(\texttt{squirrel} \mid \texttt{black}) = \frac{0+1}{C(\texttt{black}) + V}$$

- How about `black wug`?

$$P_{+1}(\texttt{black wug}) = P_{+1}(\texttt{wug} \mid \texttt{black}) = \frac{0+1}{C(\texttt{black}) + V}$$

- Would it make a difference if we used a better smoothing method (e.g., Good-Turing?)

Back-off and interpolation

The general idea is to fall-back to lower order n-gram when estimation is unreliable

- Even if,

$$C(\texttt{black squirrel}) = C(\texttt{black wug}) = 0$$

  it is unlikely that

$$C(\texttt{squirrel}) = C(\texttt{wug})$$

  in a reasonably sized corpus

Back-off

Back-off uses the estimate if it is available, 'backs off' to the lower order n-gram(s) otherwise:

$$P(w_i \mid w_{i-1}) = \begin{cases} P^*(w_i \mid w_{i-1}) & \text{if } C(w_{i-1}wi) > 0 \\ \alpha P(w_i) & \text{otherwise} \end{cases}$$

where,

- $P^*(\cdot)$ is the discounted probability
- α makes sure that $\sum P(w)$ is the discounted amount
- $P(w_i)$, typically, smoothed unigram probability

Interpolation

Interpolation uses a linear combination:

$$P_{int}(w_i \mid w_{i-1}) = \lambda P(w_i \mid w_{i-1}) + (1 - \lambda) P(w_i)$$

In general (recursive definition),

$$P_{int}(w_i \mid w_{i-n+1}^{i-1}) = \lambda P(w_i \mid w_{i-n+1}^{i-1}) + (1 - \lambda) P_{int}(w_i \mid w_{i-n+2}^{i-1})$$

- $\sum \lambda_i = 1$
- Recursion terminates with
  - either smoothed unigram counts
  - or uniform distribution $\frac{1}{V}$

Some shortcomings of the n-gram language models

The n-gram language models are simple and successful, but …

- They cannot handle long-distance dependencies:
  `In the last race, the horse he bought last year finally ______.`
- The success often drops in morphologically complex languages
- The smoothing methods are often 'a bag of tricks'
- They are highly sensitive to the training data: you do not want to use an n-gram model trained on business news for medical texts

Cluster-based n-grams

- The idea is to cluster the words, and fall-back (back-off or interpolate) to the cluster
- For example,
  - a clustering algorithm is likely to form a cluster containing words for food, e.g., `{apple, pear, broccoli, spinach}`
  - if you have never seen `eat your broccoli`, estimate

$$P(\texttt{broccoli} \mid \texttt{eat your}) = P(\texttt{FOOD} \mid \texttt{eat your}) \times P(\texttt{broccoli} \mid \texttt{FOOD})$$

- Clustering can be

hard a word belongs to only one cluster (simplifies the model)

soft words can be assigned to clusters probabilistically (more flexible)

Skipping

- The contexts
  - `boring | the lecture was`
  - `boring | (the) lecture yesterday was`

  are completely different for an n-gram model
- A potential solution is to consider contexts with gaps, 'skipping' one or more words
- We would, for example model $P(e \mid abcd)$ with a combination (e.g., interpolation) of
  - $P(e \mid abc\_)$
  - $P(e \mid ab\_d)$
  - $P(e \mid a\_cd)$
  - ...

Modeling sentence types

- Another way to improve a language model is to condition on the sentence types
- The idea is different types of sentences (e.g., ones related to different topics) have different behavior
- Sentence types are typically based on clustering
- We create multiple language models, one for each sentence type
- Often a 'general' language model is used, as a fall-back

Caching

- If a word is used in a document, its probability of being used again is high
- Caching models condition the probability of a word, to a larger context (besides the immediate history), such as
  - the words in the document (if document boundaries are marked)
  - a fixed window around the word

Structured language models

- Another possibility is using a generative parser
- Parsers try to explicitly model (good) sentences
- Parsers naturally capture long-distance dependencies
- Parsers require much more computational resources than the n-gram models
- The improvements are often small (if any)

Maximum entropy models

- We can fit a logistic regression 'max-ent' model predicting $P(w \mid \text{context})$
- Main advantage is to be able to condition on arbitrary features

Neural language models

- Similar to maxent models, we can train a feed-forward network that predicts a word from its context
- (gated) Recurrent networks are more suitable to the task:
  - Train a recurrent network to predict the next word in the sequence
  - The hidden representations reflect what is useful in the history
- Combined with *embeddings*, RNN language models are generally more successful than n-gram models
- In recent years, *masked language models*, combined with neural network architectures called Transformers became the dominant language models

Summary

- We want to assign probabilities to sentences
- N-gram language models do this by
  - estimating probabilities of parts of the sentence (n-grams)
  - use the n-gram probability and a conditional independence assumption to estimate the probability of the sentence
- MLE estimate for n-gram overfit
- Smoothing is a way to fight overfitting
- Back-off and interpolation yields better 'smoothing'
- There are other ways to improve n-gram models, and language models without (explicitly) use of n-grams

Summary

- We want to assign probabilities to sentences
- N-gram language models do this by
  - estimating probabilities of parts of the sentence (n-grams)
  - use the n-gram probability and a conditional independence assumption to estimate the probability of the sentence
- MLE estimate for n-gram overfit
- Smoothing is a way to fight overfitting
- Back-off and interpolation yields better 'smoothing'
- There are other ways to improve n-gram models, and language models without (explicitly) use of n-grams

Next:

- (?) Neural language models
- Tokenization / Computational morphology

Additional reading, references, credits

- Textbook reference: Jurafsky and Martin (2009, chapter 4) (draft chapter for the 3rd version is also available). Some of the examples in the slides come from this book.
- Chen and J. Goodman (1998) and Chen and J. Goodman (1999) include a detailed comparison of smoothing methods. The former (technical report) also includes a tutorial introduction
- J. T. Goodman (2001) studies a number of improvements to (n-gram) language models we have discussed. This technical report also includes some introductory material
- Gale and Sampson (1995) introduce the 'simple' Good-Turing estimation noted on Slide 12. The article also includes an introduction to the basic method.

Additional reading, references, credits (cont.)

- The quote from *2001: A Space Odyssey*, 'I'm sorry Dave. I'm afraid I can't do it.' is probably one of the most frequent popular culture quotes in the CL literature. It was also quoted, among many others, by Jurafsky and Martin (2009).
- The HAL9000 camera image on page 12 is from Wikipedia, (re)drawn by Wikipedia user Cryteria.

Chen, Stanley F and Joshua Goodman (1998). *An empirical study of smoothing techniques for language modeling*. Tech. rep. TR-10-98. Harvard University, Computer Science Group. URL: `https://dash.harvard.edu/handle/1/25104739`.

— (1999). "An empirical study of smoothing techniques for language modeling". In: *Computer speech & language* 13.4, pp. 359–394.

Chomsky, Noam (1968). "Quine's empirical assumptions". In: *Synthese* 19.1, pp. 53–68. DOI: `10.1007/BF00568049`.

Gale, William A and Geoffrey Sampson (1995). "Good-Turing frequency estimation without tears". In: *Journal of Quantitative Linguistics* 2.3, pp. 217–237.

Goodman, Joshua T (2001). *A bit of progress in language modeling extended version*. Tech. rep. MSR-TR-2001-72. Microsoft Research.

Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.

Additional reading, references, credits (cont.)

Shillcock, Richard (1995). "Lexical Hypotheses in Continuous Speech". In: *Cognitive Models of Speech Processing*. Ed. by Gerry T. M. Altmann. MIT Press.

blank

blank

blank

blank